Effective Lagrangian Approach to the Fermion Mass Problem

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Abstract

An effective theory is proposed, combining the standard gauge group $SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ with a horizontal discrete symmetry. By assigning appropriate charges under this discrete symmetry to the various fermion fields and to (at least) two Higgs doublets, the broad spread of the fermion mass and mixing angle spectrum can be explained as a result of suppressed, non-renormalisable terms. A particular model is constructed which achieves the above while simultaneously suppressing neutral Higgs-induced flavour-changing processes.

I. INTRODUCTION AND PHILOSOPHY

One of the most intriguing problems still outstanding with the standard model (SM) is the unexplained nature of the quark and lepton mass and mixing angle hierarchies. The range of values for the mixing angles spans some three orders of magnitude, while that for the masses spans at least six, yet in the SM these values result from Yukawa terms of the same form but with different (and unpredicted) coupling constants. It seems very unnatural for the range of values for these constants to span so many orders of magnitude. One way to explain this hierarchy is to propose that, through some as yet unknown mechanism, the heavier particle

masses and larger mixing angles are generated at lowest order in some expansion parameter, and that the particles of smaller mass receive no lowest-order contribution, only gaining masses from higher-order terms.

In this paper we outline a mechanism by which this cascade effect may be achieved. The key points are:

(1) We assume a more fundamental theory to exist at very high energies (we expect this level to be in the TeV range) which contains much more information on quark and lepton flavours than does the SM.

(2) We further assume that the effective theory below the electroweak scale ($\simeq 300$ GeV) contains the SM gauge symmetry $G_{SM} = SU(3)_C \otimes SU(2)_L \otimes U(1)_Y$ *together with an additional horizontal discrete symmetry D* [it is, of course, possible to try and explain the mass and mixing angle spectrum just using a horizontal symmetry, without recourse to effective theory. For examples of this, see (using discrete symmetries) [1] and (using continuous symmetries) [2]]. This horizontal discrete symmetry is the minimal amount of information on the flavour sector of the theory that is assumed to trickle down from the fundamental theory to our effective low-energy world. Beyond this, no further information about the high-energy theory is required nor sought.

(3) The effect of the discrete symmetry D is to put non-trivial flavour structure into the pattern of higher-dimensional, non-renormalisable operators that will contribute to fermion mass generation after electroweak symmetry breaking. The effective theory will contain terms of the form

$$\overline{f_1}f_1\phi, \quad \frac{1}{\Lambda^2}\overline{f_2}f_2\phi(\phi^\dagger\phi), \quad \frac{1}{\Lambda^4}\overline{f_3}f_3\phi(\phi^\dagger\phi)^2. \tag{1}$$

where Λ sets the scale of the new flavour physics. When ϕ gains a non-zero VEV $\langle\phi\rangle \equiv v$ the above fermions gain a hierarchical pattern of masses,

$$m_1 \sim v \gg m_2 \sim \frac{v^3}{\Lambda^2} \gg m_3 \sim \frac{v^5}{\Lambda^4} \gg \ldots \tag{2}$$

because we assume that $\Lambda \gg v$. Hierarchical mixing terms between flavours will be arranged

to also reproduce a CKM mixing angle hierarchy (a related but contrasting approach postulates the radiative generation of mass and mixing angle hierarchies. See [3] for a review).

(4) At least two Higgs doublets are required in order for operators like $\phi^\dagger \phi$ to transform non-trivially under D.

Our analysis will uncover simple candidates for the horizontal discrete symmetry D that yield a reasonable qualitative understanding of masses and mixing angles[1].

Before proceeding with our analysis, we would like to make a few more points: (i) The new flavour physics is assumed to be encoded by a new discrete symmetry, rather than a gauge or continuous global symmetry. This may be phenomenologically advantageous, because the new symmetry has to remain unbroken until the electroweak scale. This would, in general, not be tolerable for a horizontal gauge symmetry, and the breaking of a continuous global symmetry would, in general, produce troublesome Goldstone Bosons[2]. (ii) The dimensionless coefficients of the operators in Eq. (1) are assumed to be numbers of order 1. Exactly what constitutes a number of order 1 is a subjective matter. Our opinion is that any number from about 0.2 to 5 qualifies as such. (iii) Note that we assume the flavour hierarchy structure to be due to two things: the new symmetry D and the additional Higgs doublets. An effective lagrangian can have structure without these two things, but then all of this structure would be due to the unknown high-energy theory. For instance, some fundamental theory may well yield hierarchical values for the coefficients of dimension-4 $\overline{f} f \phi$ terms, thus explaining flavour. However such theories are inaccessible to us at present, so we concentrate on the alternative and more interesting possibility that the flavour information is already present

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[1]While this paper was being prepared, a paper on a similar theme appeared by Leurer, Nir and Seiberg [4]. The above paper also brought our attention to an earlier paper by Froggatt and Nielsen [5] which advocates a very similar idea.

[2]In particular models employing continuous symmetries it may, however, turn out that dangerous processes are sufficiently suppressed by small mixing angles.

at the electroweak scale. (iv) We expect flavour-changing neutral Higgs effects to exist in the effective theory. This phenomenological signature is an important generic prediction of models of our form.

The remainder of this paper is structured as follows: In Sec. II we examine the possibility that the third generation fermions (t,b,τ) gain mass from dimension-4 operators, while second generation (c,s,μ) and first generation (u,d,e) fermions gain mass from dimension-6 and dimension-8 operators respectively. Sec. III then considers a more complicated pattern wherein only the top quark gains mass at the dimension-4 level. We make some phenomenological remarks on Higgs boson physics in Sec. IV and we conclude in Sec. V.

II. THE GENERATION GAP

In this section we give a simple, warm-up example of how our mechanism is implemented and use it to propose an explanation of the most prominent trend in the mass spectrum. A quick look at the fermion masses will reveal a definite hierarchy between the three generations of particles. In each particle sector (the "up" quarks, the "down" quarks and the charged leptons), the third generation particle is consistantly heavier than the second generation particle which in turn is heavier than the first generation particle (we reserve the term "generations" here to refer to the repeated patterns of fermion quantum numbers, so that the up and down quarks and the electron and electron neutrino constitute the first generation, the charm and strange quarks and the muon and its neutrino the second generation and so on). This separation in scale is typically around two orders of magnitude per generation. For simplicity, it is assumed that the three sectors of particles will follow identical patterns of mass generation, so that only one "generic" sector need be looked at to know how all the fermions will behave (for ease of reference, this sector will be named for the charged leptons: tauon, muon and electron).

The mass Lagrangian is assumed to be made up of the following types of terms:

$$\mathcal{L}_{\text{mass}} = \lambda \overline{f}_L f_R \phi_1 + \frac{\lambda}{\Lambda^2} \overline{f}_L f_R \phi_1 (\phi_2^\dagger \phi_1) + \frac{\lambda}{\Lambda^4} \overline{f}_L f_R \phi_1 (\phi_2^\dagger \phi_1)(\phi_2^\dagger \phi_1) + \text{H.c.} \tag{3}$$

The first term is a standard, renormalisable Yukawa coupling term, and it is assumed that the heaviest particle(s) (here the tauon) will gain mass(es) from terms such as this. The remaining terms are non-renormalisable terms. The constant Λ (which has the dimensions of mass) is required to ensure that the Lagrangian density has mass dimension 4. The value of Λ is of the order of the breaking energy for the (unknown) high-energy fundamental group which is broken down to a product of the SM and some discrete group D, and thus is very large (of the order of TeVs). This will suppress these higher-order (in Λ) terms. The muon is presumed not to participate in any of the renormalisable (tree-level) terms, but to get its mass only from the non-renormalisable terms of order Λ^2 or above. The electron only combines in terms such as the third term in Eq. (3) above (and in higher-order terms), thereby receiving only a very small mass [suppressed heavily by the factor $(\frac{v}{\Lambda})^4$].

In order to assure that this can happen, the charges of the fermion fields and the Higgs fields under the discrete symmetry D are chosen so that no undesired terms (such as a tree-level, dimension-four mass term for the electron) can be found which are invariant under D while it is assumed that the effective Lagrangian is required to be invariant under the discrete symmetry.

For the current example, the simplest discrete group that will provide a mass hierarchy between the generations is Z_5. It is easy enough then to restrict the particle masses to appropriate sizes by giving them suitable charges under Z_5, however careful choices need to be made if a reasonable form for the Cabbibo-Kobayashi-Maskawa (CKM) matrix is to be obtained. A passable approximation to the CKM matrix can indeed be achieved using Z_5, but at the cost of introducing a fine-tuning condition. The CKM matrix is comprised of rotation matrices, one from each of the two quark sectors, used to diagonalise the quark mass matrices. The fine tuning condition arises in the process of determining these rotation matrices, where a very precise cancellation is required between two (a priori) large, distinct parameters in order to determine a consistant form for the rotation matrices.

Since the simplest possibility for the discrete group runs into trouble, a more complicated choice is sought. It turns out that the simplest choice of discrete group that can achieve the

same results as for the Z_5 symmetry discussed above, but with less egregious fine tuning, is the next smallest cyclic group, Z_6. Table I lists the various particle fields and the charges assigned to them under the discrete symmetry.

The only 4-dimensional term allowed by the above assignments is

$$\mathcal{L}_{4-\mathrm{d}} = \lambda_1 \overline{\tau}_L \tau_R \phi_1 + \mathrm{H.c.} \tag{4}$$

generating a mass for the tauon. The possible 6-dimensional terms are

$$\mathcal{L}_{6-\mathrm{d}} = \frac{\lambda_2}{\Lambda^2} \overline{\tau}_L e_R \phi_1 (\phi_2^\dagger \phi_1) + \frac{\lambda_3}{\Lambda^2} \overline{\mu}_L \mu_R \phi_2 (\phi_1^\dagger \phi_2) + \frac{\lambda_4}{\Lambda^2} \overline{\tau}_L \mu_R \phi_1 (\phi_2^\dagger \phi_1) +$$
$$\frac{\lambda_5}{\Lambda^2} \overline{\mu}_L e_R \phi_2 (\phi_1^\dagger \phi_2) + \mathrm{H.c.} \tag{5}$$

These terms will add small contributions to the tauon mass, generate the muon mass and some of the mixing angles. The electron mass and the rest of the mixing angles will be generated from the following 8-dimensional terms:

$$\mathcal{L}_{8-\mathrm{d}} = \frac{\lambda_6}{\Lambda^4} \overline{\mu}_L \tau_R \phi_2 (\phi_1^\dagger \phi_2)^2 + \frac{\lambda_7}{\Lambda^4} \overline{e}_L e_R \phi_2 (\phi_1^\dagger \phi_2)^2 + \frac{\lambda_8}{\Lambda^4} \overline{e}_L \mu_R \phi_2 (\phi_1^\dagger \phi_2)^2 +$$
$$\frac{\lambda_9}{\Lambda^4} \overline{e}_L \tau_R \phi_1 (\phi_2^\dagger \phi_1)^2 + \mathrm{H.c.} \tag{6}$$

There will also be very small corrections from higher order terms (dimension 10 or above).

Together, these terms will result in a mass matrix of the form

$$M = \begin{pmatrix} \mu & \mu & \mu \\ m & m & \mu \\ m & m & M \end{pmatrix}, \tag{7}$$

where $M \sim \lambda v \gg m \sim \lambda v (\frac{v}{\Lambda})^2 \gg \mu \sim \lambda v (\frac{v}{\Lambda})^4$ (note that these values are orders of magnitudes only, so that different instances of the same value may differ by small correction factors which will result from higher-order contributions and from differing Yukawa coupling constants). In order to find the mixing angles for this model, we need to find the rotation matrices that will diagonalise the above matrix. In general, two such matrices, R and L, are required (D is the diagonalised mass matrix):

$$L^\dagger M R = D. \tag{8}$$

However, only the left-hand rotation matrix, L, is needed to determine the mixing angles. If we multiply the raw mass matrix above by its hermitian conjugate, the resulting "squared" matrix will be diagonalised by L alone (which we can thus determine by studying the characteristic equation for the "squared" matrix):

$$D^2 = L^\dagger M M^\dagger L. \tag{9}$$

This "squared" matrix has the form

$$MM^\dagger = \begin{pmatrix} 3\mu^2 & \mu^2 + 2m\mu & 2m\mu + M\mu \\ \mu^2 + 2m\mu & 2m^2 + \mu^2 & 2m^2 + M\mu \\ 2m\mu + M\mu & 2m^2 + M\mu & M^2 + 2m^2 \end{pmatrix} \sim \begin{pmatrix} \mu^2 & m\mu & m^2 \\ m\mu & m^2 & m^2 \\ m^2 & m^2 & M^2 \end{pmatrix} \tag{10}$$

(noting that, in terms of order of magnitude, $M\mu \sim m^2$) and diagonalising this matrix leads to the (left-hand) rotation matrix having the form ($\epsilon \sim \frac{v}{\Lambda}$)

$$L = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^4 \\ \epsilon^4 & \epsilon^4 & 1 \end{pmatrix}. \tag{11}$$

The CKM matrix is equal to $U_L^\dagger D_L$, where U_L is the left-hand rotation matrix for the up-quark sector and D_L is the corresponding matrix for the down-quark sector. In this case, since all three sectors have been assumed to have identical Z_6 assignment schemes, these two matrices will both be of the above form, leading to a matrix with approximately 1 along the diagonals and a Cabbibo angle (the first-second generation mixing angle) larger than the other two angles, which is in qualitative agreement with observation. To further enhance this result, one could vary some of the coupling constants a little to split these latter two angles, however the simplicity of taking all three sectors to transform via the same pattern under D leads to a much more significant divergence from reality since clearly the particles of a given generation do *not* have (approximately) the same mass.

7

At this point we shall leave the simpler generation-hierarchy model we have been using in favour of a more complicated pattern of charge assignments with a view to explaining the intra-generational hierarchies and to obtain better results for the mixing angles.

III. A MORE COMPLICATED HIERARCHY

In this section we shall show how our mechanism can be used to generate a more complicated hierarchy. The previous pattern, looked at in Sec. II, had several problems with it that we would like to fix in the more complicated hierarchy to be used here. First, it failed to explain the hierarchy between the 1st-and-3rd generation mixing angle and the 2nd-and-3rd generation mixing angle. Second, it was assumed that all the masses of a given generation were of roughly the same size. To fit with reality, one needs to assume large splittings in the coupling constants to explain such hierarchies as that between the top quark and the tauon ($\frac{\lambda_{\mathrm{top}}}{\lambda_{\mathrm{tauon}}} \sim 100$). Finally, for reasonable choices for ϵ based on the observed ratios between the masses of particles in different generations, the Cabibbo angle generated by the simple hierarchy comes out far too small. The hierarchy we shall address is shown in table II, with the top quark heading the list, and ending with the electron (similar hierarchies are looked at in [6]).

The smallest suitable discrete group that we can use turns out to be Z_{13}. Reasonable results were achieved using this group with the quark particle fields having the charge assignments under the discrete symmetry shown in table III, providing only that one assumes that while the coupling constants must all be of order 1, they need not all be strictly equal to 1. Note that it is simple to generate the right magnitudes for the masses if mixing angles do not have to be considered, but that finding a suitable result for the CKM matrix *as well as* for the masses can be very difficult. Indeed, because of neutral Higgs flavour changing effects, there must also be a suitable hierarchy in the right-hand rotation matrices. It is in fact possible to obtain satisfactory masses and CKM mixing angles using a Z_9 symmetry, but in this case the right-hand rotation matrix for the down-quark sector is highly degenerate,

which leads to extreme lower limits (of the order of 5 TeV or more) on the Higgs masses (see Sec. IV).

The assignments given in table III will generate two 4-dimensional terms in the mass lagrangian, both involving the left-hand top quark field:

$$\mathcal{L}_{4-\mathrm{d}} = \lambda_{tt}\overline{t}_L t_R \phi_2 + \lambda_{tc}\overline{t}_L c_R \phi_2 + H.c. \tag{12}$$

Leaving aside any further terms involving the above combinations of fermion fields (which have negligible effect on the mass matrices due to the ϵ^2 suppression factor), the allowed dimension-6 mass terms are

$$\Lambda^2 \mathcal{L}_{6-\mathrm{d}} = \lambda_{tu}\overline{t}_L u_R \phi_2 (\phi_1^\dagger \phi_2) + \lambda_{ct}\overline{c}_L t_R \phi_2 (\phi_1^\dagger \phi_2) +$$
$$\lambda_{cc}\overline{c}_L c_R \phi_2 (\phi_1^\dagger \phi_2) + \lambda_{bb}\overline{b}_L b_R \phi_2 (\phi_1^\dagger \phi_2) +$$
$$\lambda_{\tau\tau}\overline{\tau}_L \tau_R \phi_2 (\phi_1^\dagger \phi_2) + +H.c. \tag{13}$$

Similarly, ignoring the (suppressed) repetitions of previous fermion field combinations, the dimension-8 terms are

$$\Lambda^4 \mathcal{L}_{8-\mathrm{d}} = \lambda_{cu}\overline{c}_L u_R \phi_2 (\phi_1^\dagger \phi_2)^2 + \lambda_{sb}\overline{s}_L b_R \phi_2 (\phi_1^\dagger \phi_2)^2 +$$
$$\lambda_{ss}\overline{s}_L s_R \phi_2 (\phi_1^\dagger \phi_2)^2 + \lambda_{\mu\mu}\overline{\mu}_L \mu_R \phi_1 (\phi_2^\dagger \phi_1)^2 +$$
$$\lambda_{\mu e}\overline{\mu}_L e_R \phi_1 (\phi_2^\dagger \phi_1)^2 + H.c., \tag{14}$$

the dimension-10 terms are

$$\Lambda^6 \mathcal{L}_{10-\mathrm{d}} = \lambda_{uu}\overline{u}_L u_R \phi_1 (\phi_2^\dagger \phi_1)^3 + \lambda_{bs}\overline{b}_L s_R \phi_2 (\phi_1^\dagger \phi_2) +$$
$$\lambda_{sd}\overline{s}_L d_R \phi_2 (\phi_1^\dagger \phi_2)^2 + \lambda_{db}\overline{d}_L b_R \phi_1 (\phi_2^\dagger \phi_1)^3 +$$
$$\lambda_{dd}\overline{d}_L d_R \phi_1 (\phi_2^\dagger \phi_1)^3 + H.c. \tag{15}$$

the dimension-12 terms are

$$\Lambda^8 \mathcal{L}_{12-\mathrm{d}} = \lambda_{ut}\overline{u}_L t_R \phi_1 (\phi_2^\dagger \phi_1)^4 + \lambda_{uc}\overline{u}_L c_R \phi_1 (\phi_2^\dagger \phi_1)^4 +$$
$$\lambda_{bd}\overline{b}_L d_R \phi_2 (\phi_1^\dagger \phi_2) + \lambda_{ds}\overline{d}_L s_R \phi_1 (\phi_2^\dagger \phi_1)^3 +$$

$$\lambda_{e\tau}\bar{e}_L\tau_R\phi_1(\phi_2^\dagger\phi_1)^4 + \lambda_{e\mu}\bar{e}_L\mu_R\phi_1(\phi_2^\dagger\phi_1)^4 +$$

$$\lambda_{ee}\bar{e}_Le_R\phi_1(\phi_2^\dagger\phi_1)^4 + H.c., \tag{16}$$

with the remaining three mass-matrix entries ($\overline{\tau_L}e_R, \overline{\tau_L}\mu_R$ and $\overline{\mu_L}\tau_R$) coming from dimension-14, -14 and -16 terms respectively.

In the "up" quark sector, the above mass terms lead to the (undiagonalised) mass matrix

$$M_U = \begin{pmatrix} \eta & \delta & \delta \\ \mu & m & m \\ m & M & M \end{pmatrix}, \tag{17}$$

where $M \sim \lambda v \gg m \sim \lambda v(\frac{v}{\Lambda})^2 \gg \mu \sim \lambda v(\frac{v}{\Lambda})^4 \gg \eta \sim \lambda v(\frac{v}{\Lambda})^6 \gg \delta \sim \lambda v(\frac{v}{\Lambda})^8$. After diagonalising this matrix, we get the mass eigenvalues for the top, charm and up quarks shown below:

$$m_t \sim M, \; m_c \sim m, \; m_u \sim \eta \tag{18}$$

and a left-hand rotation matrix of the form (again, $\epsilon \sim \frac{v}{\Lambda}$)

$$L_U = \begin{pmatrix} 1 & \epsilon^6 & \epsilon^8 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}. \tag{19}$$

For the "down" quark sector, we get the following mass matrix (with m, μ, and so on being of the same orders of magnitude as for the "up" quark matrix):

$$M_D = \begin{pmatrix} \eta & \delta & \eta \\ \eta & \mu & \mu \\ \delta & \eta & m \end{pmatrix}, \tag{20}$$

which leads to a left-hand rotation matrix of the form

$$L_D = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}. \tag{21}$$

and the mass eigenstates

$$m_b \sim m, \ m_s \sim \mu, \ m_d \sim \eta. \tag{22}$$

Combining these rotation matrices gives a CKM matrix with the following form

$$U_{CKM} = \begin{pmatrix} 1 & \epsilon^2 & \epsilon^4 \\ \epsilon^2 & 1 & \epsilon^2 \\ \epsilon^4 & \epsilon^2 & 1 \end{pmatrix}, \tag{23}$$

that is, the same as L_D. The charged leptons will have mass eigenstates

$$m_\tau \sim m, \ m_\mu \sim \mu, \ m_e \sim \delta. \tag{24}$$

The left-hand rotation matrix generated for the charged leptons (under these particular assignmnents) has the form

$$L_{CL} = \begin{pmatrix} 1 & \epsilon^4 & \epsilon^6 \\ \epsilon^4 & 1 & \epsilon^8 \\ \epsilon^6 & \epsilon^6 & 1 \end{pmatrix}. \tag{25}$$

These quantities will, of course, be unphysical unless the neutrinos have mass.

The mass spectrum desired above has now been achieved, but we still have an unwanted approximate degeneracy ($\theta_{12} \sim \theta_{23}$) among the quark mixing angles. It appears that this is the best one can achieve for any set of assignments and discrete groups up to at least Z_{13}.

At this point, then, in order to improve on the model, one needs to add something extra to it. One possible such addition would be to have the VEVs of the two Higgs doublets differ (although they would still be of the same order of magnitude, so that $v_1 \sim v_2 \ll \Lambda$), so that for instance contributions to the Cabibbo angle would come from particle fields coupling to the Higgs doublet with the larger VEV, while the second-third generation mixing angle would receive contributions only from fields coupled to the other Higgs doublet. For the particular choice of fermion-field assignments under the discrete symmetry given in this paper, however, this possibility proved unable to resolve the angle degeneracy while maintaining suitable values for the fermion masses.

Here, we shall look at another possibility, that of taking the assignments under Z_{13} used above and assuming that the coupling constants are of order 1, rather than strictly equal to 1. It turns out to be possible to generate a reasonable mass and mixing angle hierarchy with none of the coupling constants exceeding the range $0.3 - 3$, and with an ϵ value of 0.24.[3] A typical set of values for the coupling constants is shown below. For the "up"-quark coupling constants:

$$
\begin{pmatrix} \lambda_{uu} & \lambda_{uc} & \lambda_{ut} \\ \lambda_{cu} & \lambda_{cc} & \lambda_{ct} \\ \lambda_{tu} & \lambda_{tc} & \lambda_{tt} \end{pmatrix} = \begin{pmatrix} -1 & -0.75 & -2 \\ 2.25 & 1.25 & 1.25 \\ 2 & -3 & -1.25 \end{pmatrix} ;
\tag{26}
$$

for the "down"-quark coupling constants:

$$
\begin{pmatrix} \lambda_{dd} & \lambda_{ds} & \lambda_{db} \\ \lambda_{sd} & \lambda_{ss} & \lambda_{sb} \\ \lambda_{bd} & \lambda_{bs} & \lambda_{bb} \end{pmatrix} = \begin{pmatrix} -2 & 3 & 1 \\ -3 & 0.3 & 1.5 \\ -1 & 1 & -3 \end{pmatrix} ;
\tag{27}
$$

and for the charged leptons:

$$
\begin{pmatrix} \lambda_{ee} & \lambda_{e\mu} & \lambda_{e\tau} \\ \lambda_{\mu e} & \lambda_{\mu\mu} & \lambda_{\mu\tau} \\ \lambda_{\tau e} & \lambda_{\tau\mu} & \lambda_{\tau\tau} \end{pmatrix} = \begin{pmatrix} 2 & 0.5 & 1 \\ 0.5 & 0.8 & 1 \\ -1 & 1 & 1 \end{pmatrix} .
\tag{28}
$$

With these values, and taking $\epsilon = 0.24$, we find that the CKM matrix becomes

$$
U_{CKM} \simeq \begin{pmatrix} 0.98 & 0.20 & 0.0011 \\ 0.20 & 0.98 & 0.042 \\ 0.0072 & 0.041 & 1 \end{pmatrix}
\tag{29}
$$

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[3]One may legitimately enquire as to whether or not perturbation theory remains valid for coupling constants that are as large as about 3. A detailed partial wave unitarity calculation would be necessary to answer this question rigorously. However, experience with Yukawa coupling constants [7] suggests that values less than about 4 or 5 lie in the perturbative regime.

which is clearly a good match with reality. For the masses, taking the mass of the down quark as definite (this model generates mass ratios, rather than absolute values for the masses), the above values for the coupling constants generate the following spectrum:

$$m_u \simeq 6.0 \text{ MeV} \quad m_c \simeq 1.57 \text{ GeV} \quad m_t \simeq 104 \text{ GeV}$$

$$m_d = 9.9 \text{ MeV} \quad m_s \simeq 37.6 \text{ MeV} \quad m_b \simeq 5.5 \text{ GeV} \tag{30}$$

$$m_e \simeq 0.52 \text{ MeV} \quad m_\mu \simeq 102 \text{ MeV} \quad m_\tau \simeq 1.75 \text{ GeV}.$$

These are not the only possible sets of coupling constants that provide a spectrum like this, but there are several qualitative constraints on the coupling constant values. The nature of the resulting spectrum is more sensitive to some of the coupling constants than to others. In particular, at least one of the two dimension-4 terms involving the left-hand top quark field must have a relatively large coupling constant [e.g. λ_{tc} in Eq. (26) above], and any of the charge $-\frac{1}{3}$ coupling constants in Eq. (27) that are not shown equal to 1 are tightly constrained to the values given. In summary, we performed a coarse search through the $0.3 < |\lambda| < 3$ parameter space region, and we found no promising regions other than the one in Eqs. (26 – 28) plus perturbations around it.

The mass hierarchy resulting from this set of coupling constants is largely in good agreement with observation, with most of the results lying within the experimentally allowed ranges for the masses, and can thus be considered accurate to the level at which small corrections due to higher-dimensional terms in the mass Lagrangian could be expected to have an effect. The most obvious mismatch is with the strange quark mass which is too low by a factor of at least 3. One can alter various coupling constants in such a way as to correct this problem, but only at the expense of creating much larger discrepancies elsewhere in the spectrum. Aside from this problem, the parameter examples given in Eqs. (26 – 28) are attractive because they use only numbers that can be reasonably called "of order 1" to achieve a good mass and mixing angle spectrum, and thus we have achieved very good progress.

Finally, it should be noted that the above value for ϵ, with the VEV v around 174 GeV, implies that the high energy scale Λ for this theory is around the TeV mark, not far removed

from present-day available accelerator energies.

IV. FCNCS AND OTHER PHENOMENOLOGY

Naturally, the introduction of a new horizontal symmetry as proposed in this paper will lead to new channels for flavour changing neutral currents (FCNCs). In this section, the possible FCNCs resulting from the extra Higgs doublet will be investigated to see what constraints observational limits on such processes place on the Higgs masses and the high energy scale Λ. The severest constraints come from processes such as contributions to the $K - \overline{K}$ mass-difference and leptonic decays. In both cases, the Higgs-induced process will be suppressed by vertex factors of powers of $\frac{v}{\Lambda}$ due to the suppressed nature of light weak-eigenstate Higgs interactions, and small mixing angles if the light mass-eigenstates are first rotated into the heavier weak-eigenstates.

There are two parts to the kaon mass difference calculation in the standard model, the short-range and long-range contributions. Only the short-range contribution has been successfully determined, giving an order-of-magnitude correct result. It is assumed, therefore, that any other contributions (including the long-range contribution and any non-SM contributions) will be of the same order of magnitude. In practice, this means that such contributions are limited only by the experimental bounds on the kaon mass difference, not on the error in this value. Nevertheless, the constraint is severe. The short-range contribution to the kaon mass difference comes from the two channels shown in Fig. 1. Following Okun [8], in the standard model we have

$$\mathcal{L}_{\Delta s=2} = G_2 \overline{s_L} \gamma_\alpha (1 + \gamma_5) d \cdot \overline{s_L} \gamma_\alpha (1 + \gamma_5) d \tag{31}$$

where G_2 is given by

$$G_2 \simeq \frac{G_F^2 m_c^2}{16\pi^2} (\sin^2 \theta_c \cos^2 \theta_c) \tag{32}$$

with

14

$$G_F = \frac{g^2}{2^{\frac{5}{2}} m_W^2} = 1.165 \times 10^{-5} (\text{GeV})^{-2}, \tag{33}$$

where g is the weak coupling-constant and m_W is the W-boson mass. Measurements of the mass difference proceed via investigations of the decay products of the kaons, and so lead to the requirement that

$$G_2 f_K^2 m_K \simeq 10^{-15} \text{GeV} \tag{34}$$

where $f_K \simeq 165$ MeV and $m_K = 498$ MeV [9] are the kaon decay constant and mass, respectively. In the processes shown in Fig. 1, the Higgs mass limit can be calculated from Eq. (34) by replacing G_2 with G_H, where

$$G_H = \frac{\lambda_H^2}{m_H^2} \times \text{Mixing-angle factors} \times 2 \text{ (for the two diagrams).} \tag{35}$$

Here, m_H is the Higgs mass, and λ_H is the coupling strength of the Higgs particle to the fermion fields and is proportional to the mass of the fermions involved. The strongest coupling will be between the Higgs field and the weak b-quark eigenstate, so the mixing angles in this case turn out to be of order ϵ^4 at each vertex — factors of ϵ^2 coming in from the mixing of each of the right- and left-handed mass eigenstates (we note here that a model was found using a Z_9 discrete symmetry which could provide a satisfactory mass and mixing angle spectrum, but that the right-hand mixing angles for the down quarks were found to be highly degenerate in this model, so that the mixing angle factor involved in this calculation was too high, resulting in a Higgs mass lower limit of around 5 TeV). Taking $\lambda_H = m_b/v$, where we take m_b to be 5.5 GeV from Eq. (30) and v is the Higgs VEV and has the value 174 GeV, and (from Sec. III) $\epsilon = \frac{v}{\Lambda} = 0.24$ we therefore get

$$m_H^2 \geq \frac{\epsilon^8 m_b^2 f_K^2 m_K}{10^{-15} v^2} \text{ GeV} \tag{36}$$

which leads to

$$m_H \geq 545 \text{GeV}. \tag{37}$$

15

This figure should be taken as an order of magnitude limit only as the above calculation is not completely rigorous, but a lower limit of a few hundred GeVs is nevertheless an acceptable result.

The limits resulting from the leptonic sector are much less severe. For example, consider the lepton-number violating tau decay

$$\tau \to \mu \overline{\mu} e. \tag{38}$$

To simplify the calculation, we take the ratio of this process to the SM process

$$\tau \to \mu \overline{\nu_\mu} \nu_\tau, \tag{39}$$

which will eliminate the Lorentz factors associated with the calculation. We find, noting the current limit on the lepton-number violating decay $(\mathrm{BR}(\tau^- \to \mu^- \mu^- e^+) < 2.7 \times 10^{-5}$ [9]),

$$m_H^2 \geq \frac{\mathrm{BR}(\tau^- \to \mu^- \overline{\nu_\mu} \nu_\tau) m_W^2 \lambda_H^2 \epsilon^6}{\mathrm{BR}(\tau^- \to \mu^- \mu^- e^+) g^2} \tag{40}$$

leading to the limit

$$m_H \geq 4.7 \text{ GeV} \tag{41}$$

for $g^2 = 4\pi\alpha = \frac{4\pi}{137}$, $m_W = 80$ GeV [9], $\epsilon = 0.24$ and $\lambda_H = \frac{m_\tau}{v} \approx 0.01$. Since this value for the Higgs mass is much less constraining than that from the kaon mass-difference, experimental observations on the latter process are likely to provide the first verification (or the strongest counter-argument) to the ideas discussed in this paper.

V. CONCLUSION

We have combined effective theory with a discrete symmetry in order to better explain the observed mass and mixing angle hierarchy. An example of the method was given for the case of the rather simplistic model in which the masses of the three generations are split but masses within a given generation remain roughly equal.

16

The method was then applied to a more ambitious, and consequently more realistic, hierarchy. For the assumption that all coupling constants remain strictly equal to one, the mass hierarchy is easily explained, but problems arise in trying to generate a realistic mixing angle hierarchy. By weakening the restriction on the coupling constants so that they need only be *of order* one, a spectrum can be produced which matches reasonably well with observation. The results are compatible with a high-energy scale for the theory of the order of a TeV.

In Sec. IV, the limits placed on the Higgs mass from FCNCs were calculated, and these place a lower limit on the Higgs mass of a few hundred GeV, due to constraints coming from the neutral kaon mass difference. The constraints from leptonic FCNCs were found to be a lot less severe.

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FIGURES

FIG. 1. Higgs-induced contributions to the Kaon mass difference

TABLES

TABLE I. Charges $e^{in\frac{\pi}{3}}$ for Fermion and Higgs Fields Under Z-6 Symmetry

n	Particle Fields
3	τ_R
2	e_R, μ_R
1	ϕ_1, μ_L
0	ϕ_2, e_L
-1	
-2	τ_L

TABLE II. A More Complicated Approximate Hierarchy for the Fermion Masses

(dim 4)	t
(dim 6)	b,c,τ
(dim 8)	s,μ
(dim 10)	u,d
(dim 12)	e

20

TABLE III. Charges $e^{in\frac{2\pi}{13}}$ for Fermion and Higgs Fields Under Z-13 Symmetry

n	Particle Fields
6	ϕ_1
5	
4	
3	b_R, u_R, $(\tau, \nu_\tau)_L$
2	t_R, c_R, e_R, μ_R
1	
0	$(u, d)_L$, $(e, \nu_e)_L$
-1	
-2	s_R, τ_R, $(\mu, \nu_\mu)_L$
-3	d_R
-4	
-5	$(t, b)_L$
-6	ϕ_2, $(c, s)_L$